Global monopole surrounded by quintessence-like matter

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We present new static spherically-symmetric solutions of Einstein equations with the quintessence-like matter surrounding a global monopole. These new solutions of the coupling scalar-Einstein equations are more complicated, which depend on the parameter of equation of state $-1 < w_q < -\frac{1}{3}$. A gravitating global monopole produces a gravitational field of de Sitter kind outside the core in addition to a solid angular deficit. In the $w_q = -\frac{1}{3}$ case, we have proved that the solution cannot exist since the density of quintessence-like tends to zero if $w_q \to -\frac{1}{3}$. As a new feature, these monopoles have the outer horizon depending on both Goldstone field and quintessence-like. Since current observations constrain $-1.14 < w_q < -0.93$, new global monopoles have interesting astrophysical applications.

1. Introduction

The phase transition in the early universe could have produced different kinds of topological defects which have some important implications in cosmology [1]. Point-like defects which undergo spontaneous symmetry breaking arise in some theories and they appear as monopoles. The global monopole, which has divergent mass in flat space-time, is one of the most interesting defects. The idea that monopoles ought to exist has proven to be remarkably durable. Barriola and Vilenkin [2] first researched the characteristic of the global monopole in curved space-time or, equivalently, its gravitational effects. When one considers gravity, the linearly divergent mass of the global monopole has an effect analogous to that of a deficit solid angle plus a tiny mass at the origin. It has been shown that this small gravitational potential is actually repulsive [3]. Liebling [4] also showed that global monopoles without horizon only exist for the parameter of deficit solid angle $\epsilon < 1$, while for $\epsilon > \sqrt{3}$ no static solutions exist at all. The configurations for $1 < \epsilon < \sqrt{3}$ were called " supermassive " monopoles. Further, Li and co-workers [5] have proposed a new class of cold stars called D-stars (defect stars). One of the most important features of such stars, compared to Q-stars, is that the theory has monopole solutions when the matter field is absent, which makes D-stars behave very differently from Q-stars. Topological defects are also investigated in Friedmann-Robertson-Walker space-time [6]. It is shown that the properties of global monopoles in asymptotically de Sitter/anti-de Sitter (dS/AdS) space-time [7] and Brans-Dicke theory [8] are very different from those of ordinary monopoles. The k-field theory, in which the non-canonical kinetic terms are introduced in the Lagrangian, has been investigated in the inflation scenario and the cosmic coincidence problem. Another interesting application of k-field is topological defects, called k-defects [9]. Monopoles [10] and vortices [11] of tachyon field, which are examples of k-fields coming from string/M-theory, have also been studied.

The huge attractive force between a global monopole M and an anti-monopole $\bar{M}$ implies that the monopole over-production problem does not exist, because pair annihilation is very efficient. Barriola and Vilenkin have shown that the radiative lifetime of the pair is very short as they lose energy by Goldstone boson radiation [2]. No serious attempt has been made to develop an analytical model of the cosmological evolution of a global monopole, so one is limited to the numerical simulations of evolution [12]. In the σ-model approximation, the average number of monopoles per horizon is $N_H \sim 4$. The gravitational field of global monopoles can lead to clustering of matter, and can later evolve into galaxies and clusters. The scale-invariant spectrum of fluctuations has been given in [12]. Moreover, one can numerically obtain the CMB anisotropy $(\delta T/T)_{rms}$ patterns [13]. Comparing the theoretical value to the observed rms fluctuation, one can find the constraint of parameters in the global monopole. Therefore, the global monopole is considered to be relevant to structure formation in the early universe.

On the other hand, current observations, such as CMB (Cosmic Microwave Background) anisotropy, SNeIa (Supernovae type Ia) and large scale structure, converge on the fact that a spatially homogeneous and gravitationally repulsive energy component, referred as dark energy, accounts for about 70 % of the energy density of universe. Some heuristic models that roughly describe the observable consequences of dark energy were proposed in recent years, a number of them stemming from fundamental physics [14] and other being purely phenomenological [15]. One possibility is that the Universe is permeated by an energy density, constant in time and uniform in space. In this case, the ratio w of pressure to energy density (the equation of state) is also a constant at all time. Another possibility is that the dark energy is some kind of dynamical fluid, so that the ratio is a function $w(a)$, where a is the scale factor of universe. Whatever, w

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should be in the range of $-1 \leq w < -\frac{1}{3}$, which is the requirement of cosmic acceleration. Furthermore, the Einstein equations for the static spherically-symmetric quintessence-like surrounding a black hole[16] have been investigated. Kiselev gave the general expression for the static spherically-symmetric energy-momentum tensor of quintessence-like matter [16].

In this paper, we investigated new static spherically-symmetric solutions of Einstein equations with the quintessence-like matter surrounding a global monopole. These new solutions of the coupling scalar-Einstein equations are more complicated, which depend on the parameter of equation of state $-1 < w_q < -\frac{1}{3}$. We show that the gravitating global monopole produces a gravitational field of de Sitter kind outside the core in addition to a solid angular deficit. Furthermore, $w_q = -\frac{1}{3}$ solution cannot exist because the density of quintessence-like tends to zero as $w_q \to -\frac{1}{3}$. The new feature is the appearance of outer horizon for the case of quintessence-like matter surrounding a global monopole. Especially, we studied the $w_q = -\frac{2}{3}$ case in detail.

2. Equations of motion

We shall work within a particular model in unit $c = 1$, where a global $O(3)$ symmetry is broken down to $U(1)$. The Lagrangian density is

$$\mathcal{L} = \frac{1}{2} g^{\mu\nu} \partial_\mu \phi^a \partial_\nu \phi^a - \frac{\lambda^2}{4} (\phi^a \phi^a - {\sigma_0}^2)^2. \tag{1}$$

where ϕ^a is triplet of scalar fields, isovector index $a = 1, 2, 3$. The hedgehog configuration describing a global monopole is

$$\phi^a = \sigma_0 f(\tilde{r}) \frac{x^a}{\tilde{r}}, \quad with \quad x^a x^a = \tilde{r}^2. \tag{2}$$

so that we shall actually have a monopole solution if $f \to 1$ at spatial infinity and $f \to 0$ near the origin.

The static spherically-symmetric metric can be written as

$$ds^2 = B(\tilde{r})dt^2 - A(\tilde{r})d\tilde{r}^2 - \tilde{r}^2(d\theta^2 + \sin^2(\theta)d\varphi^2). \tag{3}$$

with the usual relation between the spherical coordinates $\tilde{r}, \theta, \varphi$ and the 'Cartesian' coordinate x^a. Introducing a dimensionless $r \equiv \sigma_0 \tilde{r}$, from (1) and (3), we obtain the equation of motion for f as

$$\frac{f''}{A} + [\frac{2}{Ar} + \frac{1}{2B}(\frac{B}{A})']f' - \frac{2}{r^2}f - \lambda^2(f^2 - 1)f = 0. \tag{4}$$

where the prime denotes the derivative with respect to r.

When we consider the static spherically-symmetric quintessence-like matter surrounding a global monopole, the Einstein equations can be written as

$$G_{\mu\nu} = 8\pi G(T_{\mu\nu} + \tau_{\mu\nu}). \tag{5}$$

where $T_{\mu\nu}$ is the energy-momentum tensor for the Lagrangian (1), and $\tau_{\mu\nu}$ is the energy-momentum tensor of the quintessence-like. $\tau_{\mu\nu}$ can be characterized by a free parameter w_q. In the static spherically-symmetric case, the general expression for the energy-momentum tensor of quintessence-like matter is given by

$$\tau_t{}^t = \tau_r{}^r = \tilde{\rho}_q, \tag{6}$$

$$\tau_\theta{}^\theta = \tau_\phi{}^\phi = -\frac{1}{2}\tilde{\rho}_q(3w_q + 1). \tag{7}$$

The whole situation concerning the energy-momentum tensor of quintessence-like matter in static spherically-symmetric case is discussed in great detail [16]. The Einstein equations (5) now are ready to be written as

$$-\frac{1}{A}(\frac{1}{r^2} - \frac{A'}{Ar}) + \frac{1}{r^2} = \epsilon^2[\frac{f^2}{r^2} + \frac{f'^2}{2A} + \frac{\lambda^2}{4}(f^2 - 1)^2 + \rho_q], \tag{8}$$

$$-\frac{1}{A}(\frac{1}{r^2} + \frac{B'}{Br}) + \frac{1}{r^2} = \epsilon^2[\frac{f^2}{r^2} - \frac{f'^2}{2A} + \frac{\lambda^2}{4}(f^2 - 1)^2 + \rho_q], \tag{9}$$

$$\frac{1}{2A}(\frac{A'}{Ar} + \frac{B'^2}{2B^2} + \frac{A'B'}{2AB} - \frac{B'}{Br} - \frac{B''}{B}) = \epsilon^2[\frac{f'^2}{2A} + \frac{\lambda^2}{4}(f^2 - 1)^2 - \frac{3w_q + 1}{2}\rho_q]. \tag{10}$$

where the small dimensionless parameter ϵ is defined by

$$\epsilon \equiv \sqrt{8\pi G {\sigma_0}^2} \approx 1.03 \times 10^{-16}(\frac{\sigma_0}{250\text{GeV}}). \tag{11}$$

and another dimensionless parameter $\rho_q \equiv \tilde{\rho}_q / {\sigma_0}^4$.

3. The solution in simplified models

Before solving the coupled equations for the metric Eqs. (8)-(10) and the scalar field Eq. (4), let us analyze an exceedingly simplified model for the monopole configuration, just to manifest the main features of the exact solution in a naive manner. The monopole configuration is simplified by a pure false vacuum inside the core, and an exactly true vacuum at the exterior [2]:

$$f = \begin{cases} 0 & \text{if } r < \delta, \\ 1 & \text{if } r > \delta. \end{cases} \tag{12}$$

The interior solution of the Einstein equations (8)-(10) can be written as follows

$$ds^2 = [1 - \frac{\epsilon^2\lambda^2}{12}r^2 + \frac{\epsilon^2\rho_0}{3w_q}r^{-3w_q-1}]dt^2 - \frac{dr^2}{[1 - \frac{\epsilon^2\lambda^2}{12}r^2 + \frac{\epsilon^2\rho_0}{3w_q}r^{-3w_q-1}]} - r^2(d\theta^2 + \sin^2\theta d\varphi^2). \quad (13)$$

where ρ_0 is the energy density of quintessence-like matter at $r = 1$. The Einstein equations outside the core are solved as follows

$$ds^2 = [1 - \epsilon^2 - \frac{2GM\sigma_0}{r} + \frac{\epsilon^2\rho_0}{3w_q}r^{-3w_q-1}]dt^2 - \frac{dr^2}{[1 - \epsilon^2 - \frac{2GM\sigma_0}{r} + \frac{\epsilon^2\rho_0}{3w_q}r^{-3w_q-1}]} - r^2(d\theta^2 + \sin^2\theta d\varphi^2). \quad (14)$$

where ϵ characterizes a deficit solid angle. We easily see that the global monopole surrounded by quintessence-like matter generates the outer horizon of de Sitter kind at $r = r_h$ if $-1 < w_q < -\frac{1}{3}$. From the continuity of the metric and its first derivatives with respect to r, we have

$$\delta = \frac{2}{\lambda}, \quad M = -\frac{16\pi\sigma_0}{3\lambda}. \quad (15)$$

Therefore, it is possible to match an interior solution to an exterior global monopole solution, but only with a negative mass M. The solution (13)-(14) is certainly not an exact one for the system of equations (4) and (8)-(10), but is an exact solution of Eqs. (8)-(10). However, we need to point out that the simplified version has revealed the principal aspects for the coupled Einstein and scalar field equations. First of all, making use of Eqs. (8)-(10) with (6) and (7), we have

$$\rho_q = \rho_0 r^{-3(w_q+1)}. \quad (16)$$

where the parameter $\rho_0 > 0$ corresponds to the density of energy of the quintessence-like matter ρ_q being positive. For $-1 < w_q < -\frac{1}{3}$, ρ_q is divergent at origin which looks surprising at first glance. There is, however, no contradiction. In fact, one should consider that the energy E_q of quintessence-like matter up to a distance R away from the origin

$$E_q = 4\pi\sigma_0 \int_0^R r^2\rho_q(r)dr = -\frac{4\pi\sigma_0\rho_0}{3w_q}R^{-3w_q}. \quad (17)$$

which is indeed finite. Secondly, the negative value (15) for M is not in conflict with Birkoff's theorem. As is known to all, Birkoff's theorem affirms that the only static spherically-symmetric vacuum solution of Einstein equations is the Schwarzschild metric, and that the parameter M in the solution is determined by the integral of $T_t^t + \tau_t^t$ along the source. Indeed, when $R > \delta$, we have, in the simplified model (12),

$$\begin{aligned} &4\pi\sigma_0[\int_0^\delta (\frac{\lambda^2 r^2}{4}) + \int_\delta^R 1 + \int_0^R (\rho_q r^2)]dr \\ &= 4\pi\sigma_0 R - \frac{4\pi\sigma_0\rho_0}{3w_q}R^{-3w_q} - \frac{16\pi\sigma_0}{3\lambda}. \end{aligned} \quad (18)$$

This quantity is really positive, and the negative constant term is just the same negative effective mass of Eq. (15). As Barriola and Vilenkin have indicated that the linear term can be understood as a kind of deficit solid angle, quite like the gravitational effects of other topological defects, such as cosmic strings and domain walls [2].

4. The solutions for coupled Einstein and scalar field equations

4.1. Effective mass

Now our attention transfers from the simplified version to the exact solution for the system to rigorously confirm the features which has been born of simplified one. For convenience, we choose the ansätz as follows

$$A^{-1}(r) = 1 - \epsilon^2 + \frac{\epsilon^2\rho_0}{3w_q}r^{-3w_q-1} - \frac{2G\sigma_0 M_A(r)}{r}. \quad (19)$$

where $-1 < w_q < -\frac{1}{3}$. Note that for $\rho_0 = 0$, the existence of solutions without horizon is restricted by $\epsilon < 1$ [13]. Eqs. (8)-(10) can be formally solved by

$$A^{-1}(r) = 1 - \frac{\epsilon^2}{r}\int_0^r [\frac{f^2}{r^2} + \frac{f'^2}{2A} + \frac{\lambda^2}{4}(f^2-1)^2 + \rho_q]r^2 dr. \quad (20)$$

and

$$B(r) = \frac{1}{A(r)} \exp{[\epsilon^2 \int_\infty^r f'^2 r dr]}. \tag{21}$$

As $r \to \infty$, we have $B(r) = A^{-1}(r)$. From Eqs. (19) and (20), we obtain

$$\begin{aligned} M_A(r) = \quad & 4\pi\sigma_0 \int_0^r [\frac{f^2}{r^2} + \frac{f'^2}{2A} + \frac{\lambda^2}{4}(f^2-1)^2 + \rho_q] r^2 dr \\ & -4\pi\sigma_0 r - \frac{4\pi\sigma_0\rho_0}{3w_q} r^{-3w_q}. \end{aligned} \tag{22}$$

which is the effective mass function of global monopole. A global monopole solution should have $\lim_{r\to\infty} f = 1$. If this convergence is quick enough in flat space, $M_A(r)$ will also converge fast to a finite value. The effective mass of the solution is determined by the asymptotic value $\lim_{r\to r_h} M_A(r) \equiv M_A$. Numerical calculations show that the shape of $f(r)$ is quite insensitive to ϵ^2 in the range $0 \leq \epsilon \leq 1$, therefore the above discussion is consistent. In order to solve the system of equations uniquely, we need to introduce boundary conditions as follows

$$f(0) = 0, A^{-1}(r_h) = B(r_h) = 0, A(0) = B(0) = 1. \tag{23}$$

4.2. A special example : $w_q = -\frac{2}{3}$ monopole

As a characteristic example, we are going to investigate $w_q = -\frac{2}{3}$ global monopole. Let us first discuss the asymptotic behavior at the origin. Expanding the functions we have

$$\begin{aligned} & f(r \ll 1) = f_1 r + \frac{3}{8}\epsilon^2 f_1 \rho_0 r^2 + O(r^3), \rho_q(r \ll 1) = \rho_0 r^{-1} + O(r^3), \\ & A^{-1}(r \ll 1) = 1 - \frac{\epsilon^2\rho_0}{2} r - \frac{1}{12}(\epsilon^2\lambda^2 + 6\epsilon^2 f_1^2) r^2 + O(r^3), \\ & B(r \ll 1) = 1 - \frac{\epsilon^2\rho_0}{2} r - \frac{1}{12}\epsilon^2\lambda^2 r^2 + O(r^3). \end{aligned} \tag{24}$$

where f_1 is a free parameter to be determined numerically. The asymptotic behavior ($r \gg 1$) is given by

$$\begin{aligned} & f(r \gg 1) = 1 - \frac{1}{\lambda^2} r^{-2} + \frac{2\epsilon^2 - 3}{2\lambda^4} r^{-4} - [\frac{\epsilon^2\rho_0(2\epsilon^2-3)}{\lambda^6} + \frac{2M_A}{\lambda^4}] r^{-5} + O(r^{-6}), \\ & A^{-1}(r \gg 1) = 1 - \epsilon^2 - \frac{\epsilon^2\rho_0}{2} r + M_A r^{-1} - \frac{\epsilon^2}{\lambda^2} r^{-2} - \frac{\epsilon^4\rho_0}{2\lambda^4} r^{-3} + O(r^{-4}), \\ & B(r \gg 1) = 1 - \epsilon^2 - \frac{\epsilon^2\rho_0}{2} r + M_A r^{-1} - \frac{\epsilon^2}{\lambda^2} r^{-2} + O(r^{-4}), \\ & \rho(r \gg 1) = \rho_0 r^{-1} + O(r^{-3}). \end{aligned} \tag{25}$$

where the effective mass $M_A = \lim_{r\to r_h} M_A(r)$ is determined numerically.

To go beyond the simplified model, we find out the gravitational field of global monopole by numerically solving the combined Einstein's and Goldstone field equations. In all numerical calculations, we can choose $\lambda = 1$ without loosing universality. The resulting solutions for $f(r)$ and $\rho(r)$ are shown in Fig. 1, where the parameters are chosen as $\epsilon = 10^{-2}$, $w_q = -\frac{2}{3}$ and $\rho_0 = 0, 0.1, 1$. It is easily found that the solution for the Goldstone scalar field $f(r)$ is not much different from the one with $\rho_0 = 0$. In the $\rho_0 = 0$ case, Eqs. (4) and (8)-(10) are reduced to the case previously studied in great detail in [2, 3, 4]. We have repeated the calculations and confirmed the previous results. From Fig. 1, we find that the shape of the curve $f(r)$ is quite insensitive to the value of ρ_0 in the interval $0 \leq \rho_0 \leq 1$ not only asymptotically, but also close to the origin. Notice that the shape of $f(r)$ is also insensitive to the value of ϵ in the interval $0 \leq \epsilon \leq 1$. However, the shape of $f(r)$ is sensitive if $w_q \neq -\frac{2}{3}$ and $\rho_0 > 1$ (see Figs. 3-4).

Using Eq. (14) we see that the global monopole surrounded by quintessence-like matter generates the outer horizon of de Sitter kind at $r = r_h$ if $-1 < w_q < -\frac{1}{3}$. Es-

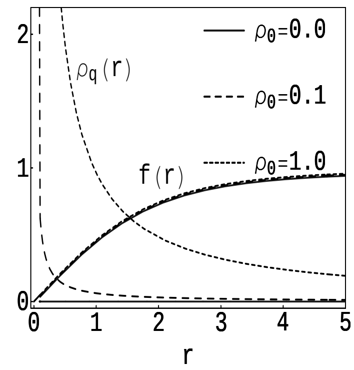

FIG. 1: The functions $f(r)$ and $\rho(r)$ are plotted vs the dimensionless coordinate r for different values of ρ_0 in the interval $0 \leq \rho_0 \leq 1$. The parameters are chosen as $\epsilon = 10^{-2}$ and $w_q = -\frac{2}{3}$. The shape of the curve $f(r)$ is quite insensitive to the value of ρ_0. However, it is not universal character for the cases with $w_q \neq -\frac{2}{3}$.

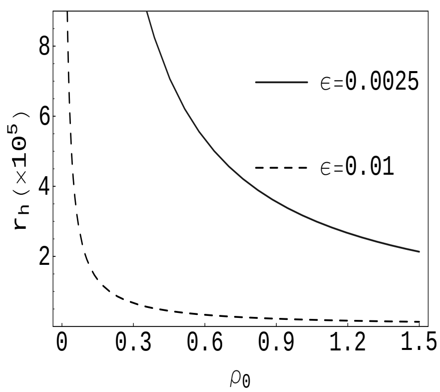

FIG. 2: The value of the radial dimensionless coordinate, where $A^{-1}(r = r_h) = 0$ is shown as function of ρ_0 for two different values of ϵ and $\lambda = 1$.

pecially, the outer horizon $r_h = \frac{1-\epsilon^2}{\epsilon^2\rho_0} + \sqrt{\frac{(1-\epsilon^2)^2}{\epsilon^4\rho_0^2} + \frac{8}{3\lambda\rho_0}}$ for $w_q = -\frac{2}{3}$ monopole in the simplified model. In the realistic model, we need to have the aid of numerical calculation. To investigate the appearance of horizon in the $w_q = -\frac{2}{3}$ monopole, we have fixed $\epsilon = 0.0025$ and 0.01, respectively, and studied the value of zero of $A^{-1}(r)$, r_h with $A^{-1}(r_h) = 0$ in dependence on ρ_0. Our results have been shown in Fig. 2. In the limit $\rho_0 \to 0$, the horizon tends to infinity. Obviously, r_h is a decreasing function of ρ_0. For increasing ρ_0, the value of r_h tends to zero as is demonstrated in Fig. 2.

4.3. The limit of $w_q \to -\frac{1}{3}$

It is worthwhile to note that $w_q = -\frac{1}{3}$ is a special case. Its asymptotic behavior must be discussed independently. In the simplified model, the interior and exterior solutions are reduced to, respectively

$$ds^2 = [1 - \epsilon^2\rho_0 - \frac{\epsilon^2\lambda^2}{12}r^2]dt^2 - \frac{dr^2}{[1 - \epsilon^2\rho_0 - \frac{\epsilon^2\lambda^2}{12}r^2]} - r^2(d\theta^2 + \sin^2\theta d\varphi^2). \quad (26)$$

and

$$ds^2 = [1 - \epsilon^2(1 + \rho_0) - \frac{2G\sigma_0 M}{r}]dt^2 - \frac{dr^2}{[1 - \epsilon^2(1 + \rho_0) - \frac{2G\sigma_0 M}{r}]} - r^2(d\theta^2 + \sin^2\theta d\varphi^2). \quad (27)$$

In the limit of $r \to \infty$, Eq. (27) implies that the metric component $|g_{rr}|$ tends to a constant value $[1 - \epsilon^2(1 + \rho_0)]^{-1}$. Therefore, we can rescale the definition of radius by $r \to r/\sqrt{|g_{rr}|}$ so that the sphere surface gets the deficit solid angle since its area becomes $4\pi r^2/|g_{rr}|$ instead of $4\pi r^2$. Certainly, Eq. (27) is not an exact solution for the metric around a $w_q = -\frac{1}{3}$ global monopole. Unfortunately, the exterior solution Eq. (27) is not consistent with the principal features of the realistic case $w_q = -\frac{1}{3}$, as we shall confirm in the text below.

Indeed, we can rigorously prove the non-existence of $w_q = -\frac{1}{3}$ global monopole by *reductio ad absurdum*. If there exists an exact monopole solution with $w_q = -\frac{1}{3}$, we have $f(0) = 0, f'(0) \neq 0, f''(0) = 0$ and $\rho_q > 0$. In the neighbor of origin, the coupled Einstein and scalar field equations can be reduced to

$$\frac{f'}{A} = \frac{f}{r}, \quad (28)$$

$$1 - \frac{1}{A} = \epsilon^2(f^2 + \rho_q r^2), \quad (29)$$

$$\frac{A'}{A} = -\frac{B'}{B}. \quad (30)$$

From Eq. (28), we have $f = f_1 r^{A(0)}$, where f_1 is an integral constant. Since $f'(0) \neq 0$ and $f''(0) = 0$, we give $A(0) = 1$ and $f_1 = f'(0)$. Using this result, Eq. (29) can be written as

$$f_1^2 + \rho_q = 0. \quad (31)$$

which conflicts with the condition $\rho_q > 0$.

Note that we have employed the Lagrangian density (1) in view of the above proof, where the Goldstone field and quintessence-like matter are assumed to interact only through their gravitational influence without direct interaction. Although non-existence of $w_q = -\frac{1}{3}$ global monopole was rigorously proved, the global monopole may exist if we introduce an interactive term in the Lagrangian density (1).

4.4. Asymptotic behavior

In this subsection, we discuss the asymptotic behavior of global monopoles for $-1 < w_q < -\frac{1}{3}$. At the

origin, the asymptotic behavior is

$$\begin{aligned} f(r \ll 1) &= f_1 r + O(r^{-3w_q}), \\ A(r \ll 1) &= 1 - \frac{\epsilon^2 \rho_0}{3w_q} r^{-3w_q-1} + O(r^{\beta}), \\ B^{-1}(r \ll 1) &= 1 - \frac{\epsilon^2 \rho_0}{3w_q} r^{-3w_q-1} + O(r^{\beta}), \\ \rho_q(r \ll 1) &= \rho_0 r^{-3w_q-3} + O(r^{\gamma}). \end{aligned} \tag{32}$$

where

$$\beta = \begin{cases} 2 & \text{if } -1 < w_q < -\frac{2}{3}, \\ -6w_q - 2 & \text{if } -\frac{2}{3} < w_q < -\frac{1}{3}. \end{cases} \tag{33}$$

and

$$\gamma = \begin{cases} -3w_q - 1 & \text{if } -1 < w_q < -\frac{2}{3}, \\ -9w_q - 5 & \text{if } -\frac{2}{3} < w_q < -\frac{1}{3}. \end{cases} \tag{34}$$

The asymptotic behavior ($r \gg 1$) is given by

$$\begin{aligned} &f(r \gg 1) = 1 - \frac{1}{\lambda^2} r^{-2} - \frac{\epsilon^2 \rho_0 (2 + 3w_q)}{3w_q \lambda^4} r^{-3w_q-5} + O(r^{-6w_q-8}), \\ &A^{-1}(r \gg 1) = 1 - \epsilon^2 + \frac{\epsilon^2 \rho_0}{3w_q} r^{-3w_q-1} + \frac{2G\sigma_0 M_A}{r} + O(r^{-2}), \\ &B(r \gg 1) = 1 - \epsilon^2 + \frac{\epsilon^2 \rho_0}{3w_q} r^{-3w_q-1} + \frac{2G\sigma_0 M_A}{r} + O(r^{-2}), \\ &\rho(r \gg 1) = \rho_0 r^{-3w_q-3} + O(r^{3w_q-1}). \end{aligned} \tag{35}$$

4.5. Numerical results

In this subsection, we still choose $\lambda^2 = 1$. As an example of $w_q \neq -\frac{2}{3}$, the resulting solution for $f(r)$ is shown in Fig. 3, where the parameters are chosen as $\epsilon = 10^{-2}, w_q = -\frac{1}{2}$ and $\rho_0 = 0, 10^2, 10^3$. It is easily found that the solution for the Goldstone scalar field $f(r)$ is sensitive to the value of ρ_0. It is inevitable that the disparity of $f(r)$ affects the value of $M(r)$. The resulting solutions of $f(r)$ are shown in Fig. 4 via different w_q, where other parameters are chosen as $\epsilon = 0.01$ and $\rho_0 = 10^3$. Notice that the shape of $f(r)$ is sensitive to the value of w_q except asymptotical region. Obviously, the less the value of w_q is, the more slowly the curve of $f(r)$ tends to unit. The global monopole surrounded by quintessence-like matter generates the horizon of de Sitter kind at $r = r_h$ if $-1 < w_q < -\frac{1}{3}$. In Fig. 5, the value of the radial dimensionless coordinate, where $A^{-1}(r = r_h) = 0$ is shown as a function of w_q for two different values of ϵ and $\lambda = 1$. It is interesting to find that all the outer horizons tend to infinite as $w_q \to -\frac{1}{3}$ in figure (5b). In Fig. 6, we plot $M(r)/\epsilon^2$ for different values of ρ_0, where $M(r) \equiv M_A(r)/4\pi\sigma_0$, the dimensionless parameter of the effective mass. Clearly, the mass function is negative for all r. Especially, the shape of these curves is considerably varied. In Fig. 7, we display $M(r)/\epsilon^2$ for different values of w_q. Obviously, the mass function is negative for all r, the value of which increases with w_q.

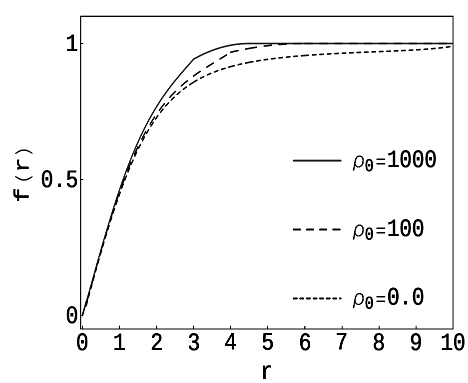

FIG. 3: The functions $f(r)$ are plotted vs the dimensionless coordinate r for different values of ρ_0. The parameters are chosen as $\epsilon = 10^{-2}, w_q = -\frac{1}{2}$ and $\lambda = 1$. The shape of the curve $f(r)$ is sensitive to the value of ρ_0 except asymptotical region.

5. Discussion and conclusions

Topological defects are considered to be relevant to structure formation in the early universe. Global defects are of special interest in this context since they have a long-range scalar field. This leads to the divergent mass in flat space, but renders a strong gravitational effect when the global defects are investigated in

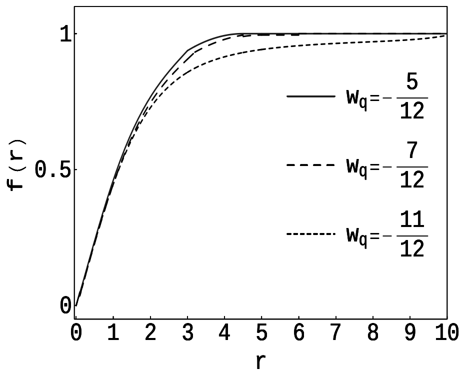

FIG. 4: The functions $f(r)$ are plotted vs the dimensionless coordinate r for different values of w_q. The parameters are chosen as $\epsilon = 10^{-2}, \rho_0 = 10^3$ and $\lambda = 1$. The shape of the curve $f(r)$ is sensitive to the value of w_q except asymptotical region.

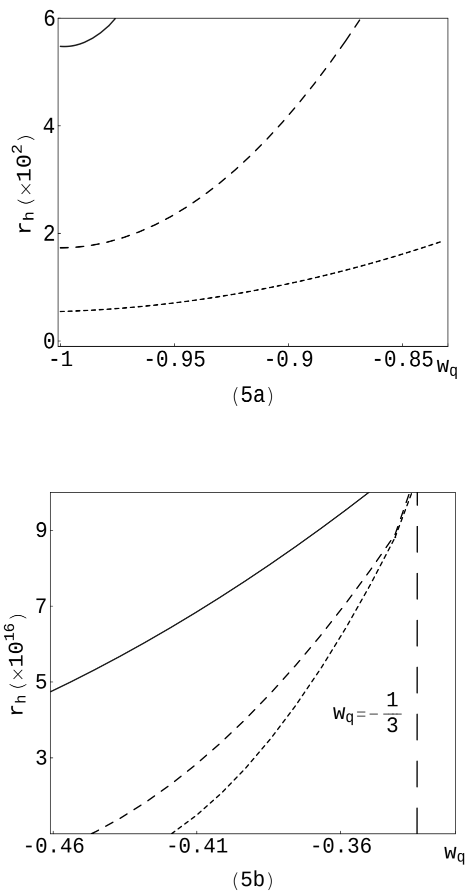

FIG. 5: The horizon r_h is shown as a function of w_q for different values of ρ_0, where solid line, long dash line and short dash line correspond to $\rho_0 = 0.1, 1, 10$ respectively. In these figures, r_h is plotted in $-1 \leq w_q \leq -0.83$ and $-0.46 \leq w_q < -\frac{1}{3}$, respectively. All the outer horizons tend to infinite as $w_q \to -\frac{1}{3}$.

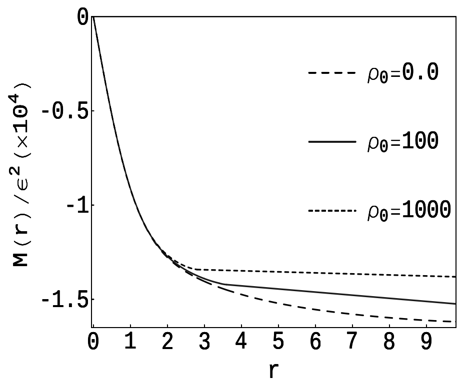

FIG. 6: The mass function $M(r)/\epsilon^2$ $(M(r) \equiv M_A(r)/4\pi\sigma_0)$ is plotted for different values of ρ_0. The other parameters are $\epsilon = 10^{-2}$, $w_q = -\frac{1}{2}$ and $\lambda = 1$.

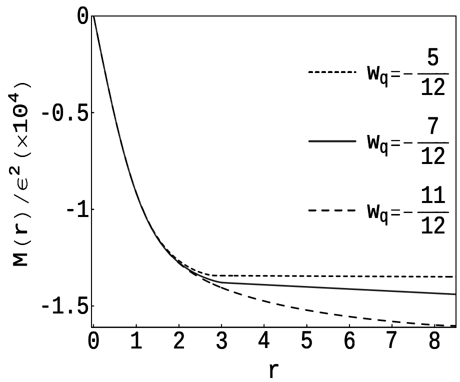

FIG. 7: The mass function $M(r)/\epsilon^2$ $(M(r) \equiv M_A(r)/4\pi\sigma_0)$ is plotted for different values of w_q. The other parameters are $\epsilon = 10^{-2}$, $\rho_0 = 10^3$ and $\lambda = 1$

curved space. In the global monopole case, the linearly divergent mass has an effective analogous to that of a deficit solid angle plus a tiny mass at the origin. In this paper, we have got the solutions of Einstein-scalar coupling equations for the static spherically-symmetric quintessence-like matter surrounding a global monopole, in which the scalar field and quintessence-like matter are assumed to interact only through their gravitational influence without direct interaction.

In a fundamental sense the stability of global monopole arises from the nontrivial topology of the vacuum manifold $\mathcal{M}$. The removal of the defect entails the infinite cost of varying all of the field hence the defect is classically stable. However, the global monopole was created in the early universe therefore it might possibly form thin shell like wormhole during phase transitions. Rahaman and colleagues [18] have researched the thin shell wormhole in the context of global monopole for Barriola and Vilenkin solution [2]. They have obtained the time evolution equation of the radius a of wormhole throat. When the initial velocity $\dot{a} = 0$, the radius is a

position of static equilibrium. They have also analyzed the dynamical stability of the thin shell, considering linearized radial perturbations around static solution of wormhole [18]. By applying this method, we can obtain a restriction on the model parameters. In our models, the evolution equation of throat can be written as

$$\dot{a} + V(a) = 0 \tag{36}$$

$$V(a) = V_{\mathbf{BV}}(a) - \frac{\epsilon^2\rho_0}{2}a^{-3w_q-1} \tag{37}$$

where $V_{\mathbf{BV}}(a)$ is the potential for Barriola-Vilenkin case [18]. By the linearization, the stability of equilibrium configurations corresponds to the condition $V''(a_0) > 0$, where the prime denotes derivative with respect to a. We have

$$V''(a) = V''_{\mathbf{BV}}(a) - \frac{1}{2}(3w_q+1)(3w_q+2)\epsilon^2\rho_0 a^{-3w_q-3} \tag{38}$$

for global monopole surrounded by quintessence-like matter. In the $-\frac{2}{3} < w_q < -\frac{1}{3}$ case, we have $V''(a) > V''_{\mathbf{BV}}(a)$ so that the restriction of parameters could be loosened; in the $-1 < w_q < -\frac{2}{3}$ case, we have $V''(a) < V''_{\mathbf{BV}}(a)$ therefore the restriction becomes tighter; in the $w_q = -\frac{2}{3}$ case, the restriction is the same as Barriola-Vilenkin case.

Our results are mainly as follows:

(i) The global monopole solutions are possible under the appropriate choice of internal parameter in the energy-momentum tensor of quintessence-like matter, depending on the parameter of equation of state $-1 < w_q < -\frac{1}{3}$.

(ii) In the $w_q = -\frac{1}{3}$ case, there exists the solution of the static spherically-symmetric quintessence-like matter surrounding a black hole. But the static spherically-symmetric quintessence-like matter surrounding a global monopole cannot exist in the $w_q \to -\frac{1}{3}$ limit, because $\rho_q \to 0$ if $w_q \to -\frac{1}{3}$.

(iii) The distinguishing feature is the appearance of outer horizon for the case of quintessence-like matter surrounding a global monopole. The numerical results show that the radius of horizon becomes larger and larger as the density of quintessence-like matter decreases.

(iv) In $w_q = -\frac{2}{3}$ case, Eq. (25) indicates that ρ_0 is a higher order effect in contrast with $w_q \neq -\frac{2}{3}$ case. Therefore, the basic features of $w_q = -\frac{2}{3}$ monopoles are similar to ordinary global monopole if we take $0 < \rho_0 \leq 1$. However, the features of new global monopole are considerably varied in $w_q \neq -\frac{2}{3}$ case. Since current observations constrain $-1.14 < w_q < -0.93$ [17], new global monopoles have interesting astrophysical applications.

Acknowledgments

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